Multilane simulations of traffic phases

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The optimal velocity model, as modified by the author, is used in simulations of traffic on a dual-lane highway and a single-lane highway with an on-ramp. The equilibrium solutions of the modified model cover a two-dimensional region of flow-density space beneath the fundamental-diagram curve, rather than just lying on the curve as in the original model. Thus it satisfies a requirement of the three-phase model of Kerner [Phys. Rev. Lett. **81**, 3797 (2002)]. Synchronization of velocity across dual lanes due to frequent lane changes is observed in free flow. True synchronized flow, as determined by the region of density-flow space it occupies, is obtained in on-ramp simulations with typical driver reaction times. A gradual change to the formation of a jam is observed for increasing delay times.

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I. INTRODUCTION

The rationale for physicists to study traffic has been given in the thorough review of the field by Helbing [1]. Observations of self-organization, nonequilibrium phase transitions, and complexity in traffic have motivated extensive work from the viewpoint of statistical mechanics. The availability of detailed highway data, as compiled for example by Kerner [2], provides opportunities to compare theoretical concepts to the real world. From a practical standpoint, the economic and social cost of congestion continues to grow, which suggests that further work to improve traffic flow is needed.

According to Kerner and Rehborn [2–5], experimental studies of traffic on a German autobahn reveal three phases: (1) free flow (FF), (2) synchronized flow (SF), and (3) the wide moving jam (WMJ) phase. FF occurs at low traffic density while both SF and WMJ occur above a critical vehicle density. SF gets its name from the synchronization of vehicle velocity on different lanes of a multilane road. Frequent lane changes by vehicles equilibrate the velocities, which normally would be higher in the fast lane compared to the slow lane(s). SF generally occurs at a highway bottleneck, such as an on-ramp. The downstream edge of this congested flow is pinned at the bottleneck whereas the upstream edge moves upstream as the congested region grows.

WMJ differs from SF in that no bottleneck is required to transform flow into this phase. A fluctuation in the lead vehicle speed can induce a jam in the following vehicles if the headway between vehicles is small enough [6]. The upstream front moves upstream at a characteristic velocity that is somewhat faster than the downstream front so that the width of the jam grows. Average velocity within a jam is quite small.

Although transitions from FF to WMJ can be observed in traffic data, Kerner reported that often the transition is more complex—FF to SF to WMJ. The dynamics of these transitions have been theoretically examined. The first simulation of SF transitions was reported by Helbing and Treiber [7], who used a gas-kinetic-based macroscopic model. Simulations with discrete-vehicle models have also reproduced SF. These include the microscopic model of Kerner and Klenov [8] and the intelligent driver model of Trieber, Hennecke, and Helbing [9]. Metastable states at high densities with flow rates obtainable in free flow at low densities have been found in cellular automaton (CA) simulations [10,11] and in some other discrete-vehicle models [12], providing further theoretical support for the concept of SF. Also, see Ref. [13], where FF-SF-WMJ transitions have been simulated with discrete vehicle and macroscopic models.

It does not appear that synchronization of the vehicle velocity is the necessarily the most important characteristic of SF. Rather it is the metastability of flow and the region of the two-dimensional vehicle density-flow rate space in which it occurs that is significant. Since SF is essential to a complete understanding of traffic flow, it is desirable to study this phase further.

The purpose of the present work is to use discrete-vehicle simulations to explore the FF to SF phase transitions in traffic flow. The advantage of using a discrete-vehicle model, as opposed to CA models, is that parameters can be related to physical quantities more directly. These include the sometimes-overlooked delay times due to driver reaction times. The optimal velocity model of Bando *et al.* [14] modified to include driver reaction times is employed here. An additional modification to eliminate unphysical oscillations in vehicle velocities is made, as described by Davis [15]. The latter modification replaced the optimal velocity function $V(\Delta x)$, where Δx is the headway to the preceding vehicle, with the velocity of the preceding vehicle in certain conditions of acceleration. This replacement changed the nature of the model from one whose equilibrium solutions lie on the fundamental diagram (a curve of flow q versus density ρ) to a model that satisfies a postulate of three-phase traffic theory [4,5]. In this theory equilibrium solutions can be found essentially anywhere in the two-dimensional region of flow-density space beneath the curve of $q=\rho V(\Delta x)$ as a function of $\rho=1/\Delta x$. Thus the modified optimal velocity (MOV) model differs qualitatively from the original model of Bando *et al.* [14] and from those described in [1].

A car-following model without explicit connection to vehicle headway was not considered for this study because ve-

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hicle density is one factor determining traffic phase. Furthermore, the inertial car following model of Tomer *et al.* [12], which does depend on headway but has unphysical velocity oscillations, was not considered. Nor was the intelligent driver model of Treiber *et al.* [9] because it does not depend upon driver reaction time. The MOV model used in the present simulations is appealing due to its simplicity and straightforward interpretation of its fundamental parameters.

The paper is organized as follows. In Sec. II, the MOV model is described. Rules for lane changing and a demonstration of velocity synchronization in a dual-lane highway are given in Sec. III. Section IV is devoted to traffic merging into a single-lane highway from an on-ramp. The transition from FF to the single-lane equivalent of SF is studied. Further discussion and conclusions are given in Sec. V.

II. DESCRIPTION OF THE MODIFIED OPTIMAL VELOCITY MODEL

Bando *et al.* [14] introduced an intriguing, simple model to describe traffic flow—the optimal velocity (OV) model. Recently it has been shown that the OV model has a direct connection to CA [16]. In this model each vehicle attempts to maintain a velocity determined by the optimal velocity function (OVF) $V(\Delta x)$, where Δx is the headway to the vehicle in front (center-to-center distance). The dynamics is first order and is characterized by a time constant τ.

Delay time t_d due to driver reaction time can be introduced by evaluating Δx at $t-t_d$. For small delays, it has been suggested that t_d could be combined with τ to renormalize the first-order time constant to $\tau'=\tau+t_d$ and eliminate the explicit dependence on delay. Typically $\tau=0.5$ s, but realistic delay times are 0.75–1.25 s. Thus it was shown that renormalization is not a good approximation and that modifications were required [17]. The modified version consisted of two changes [15]. First the headway variable Δx in the OVF was replaced by $\Delta x(t-t_d)+t_d[\Delta v(t-t_d)]$ where Δv is the difference in velocities (the time rate of change of headway). Second, partial car following was introduced for acceleration by replacing $V(\Delta x(t-t_d)+t_d[\Delta v(t-t_d)])$ with the velocity of the preceding vehicle (evaluated at $t-t_d$) if it is smaller. These replacements eliminated frequent vehicle collisions (headway less than the vehicle length) and unphysical oscillations in vehicle velocity. The related notion that, within some interval of velocity and headway, a vehicle adopts the velocity of the vehicle it follows has been suggested in Refs. [8,11]. An additional benefit accrues for simulations. Initial, stable conditions can be prepared with headways larger than the equilibrium headways determined by the OVF. Expanded headways can be maintained if vehicle velocities remain the same (typical of car following behavior). The appropriate equations for the nth vehicle are

$$\tau\frac{dv_n(t)}{dt}+v_n(t)=V_{desired}, \tag{1}$$

where

$$V_{desired}=V_{OV} \quad [V_{OV}<v_n(t)] \tag{2a}$$

$$=\min\{V_{OV},v_{n-1}(t-t_d)\} \quad [V_{OV}>v_n(t)], \tag{2b}$$

and

$$V_{OV}=V(\Delta x_n(t-t_d)+t_d\Delta v_n(t-t_d)), \tag{3}$$

where

$$V(\Delta x_n)=V_0\{\tanh[C_1(\Delta x_n-\Delta x^0)]+C_2\},$$
$$\Delta x_n=x_{n-1}-x_n. \tag{4}$$

Note that $v_n(t)$, not $v_n(t-t_d)$, appears in Eqs. (1) and (2). The reason for this choice (which differs from that of Ref. [18]) is that I regard the dynamics as first-order control of the vehicle velocity to the desired velocity, which is determined by a delayed human response. Further, since the model requires, when the vehicle accelerates, switching from the OVF to the velocity of the preceding vehicle (unless it is too high), it is $v_n(t)$, not $v_n(t-t_d)$, that determines this condition.

Throughout this paper, $\tau=0.5$ s and the OVF $V(\Delta x)$ is taken to be the parametrization given by Sugiyama [19]: $C_1=0.086$/m, $C_2=0.913$, $\Delta x^0=25$ m, and $v_0=16.8$ m/s. Except where noted, $t_d=0.75$ s. All vehicles are identical.

III. DUAL-LANE MODEL

In this section, a dual-lane highway model is presented. The fast lane is labeled lane 1. Rules for changing lanes are diagrammed in Fig. 1. The rules are symmetric: that is, the rules to change from lane 1 to lane 2 are the same as from lane 2 to lane 1. If the closest vehicle in front of it is in the same lane and the closest following vehicle is also in the same lane (at $t-t_d$), then the candidate is permitted to change lanes (at t); if the following vehicle is in the other lane, but the headway b is larger than the safe distance b_{safe}, a lane change is still permitted. The safe distance is determined by the OVF and the following vehicle velocity $u=V(b_{safe})$. (For consistency, v and b are evaluated at $t-t_d$.) Since the OVF is monotonic, b_{safe} can be uniquely determined. The first simulation vehicle in each lane follows the same lead vehicle, which cannot be passed. The lead vehicle starts at $x=0$ at $t=0$ and travels with a specified velocity profile. Lane changes are allowed every 0.05 s and candidates are chosen randomly. On average, each vehicle is a candidate every 0.05 s. A maximum of 600 vehicles are used in the simulations.

The initial positions and velocities of the simulation vehicles are given as follows. (Note this is an open system with no period boundary conditions.) In lane 1, the position of the jth vehicle site is $x=-j\ h_1\ e_1$ and the velocity is $v=V(h_1)$, where h_1 is the headway and e_1 is the expansion factor. The probability that a site is occupied with a vehicle is p_1. Lane 2 is of the same form with h_2, e_2, and p_2. The vehicles are numbered from 1 to N according to their initial order regardless of lane, car 1 being first. Over time vehicles may change lanes and order, but their car number remains

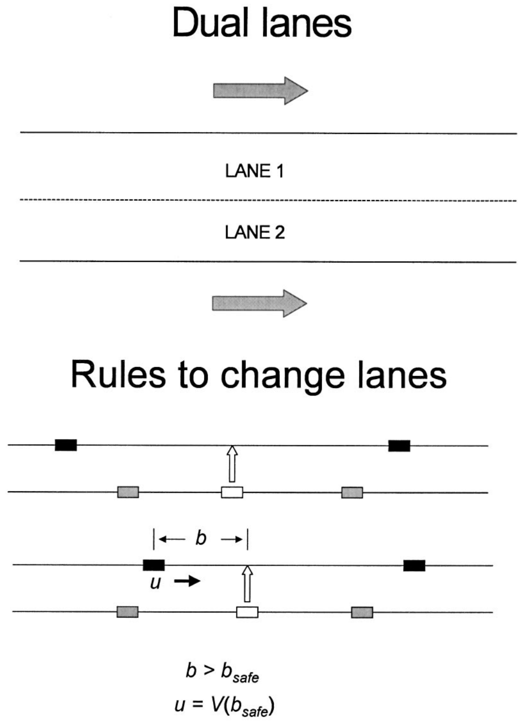

FIG. 1. Rules to change lanes in dual-lane model are symmetric for lanes 1 and 2. If the headways to the vehicles in the same lane preceding and following the candidate vehicle are each less than the headways to the corresponding vehicles in the other lane, a lane change is permitted. If only the headway to preceding vehicle is less, a lane change is still permitted if the headway b to the following vehicle in the opposite lane exceeds the safe headway b_{safe} determined by the optimal velocity function and the following vehicle velocity. All quantities are evaluated at the delayed time $t - t_d$.

fixed. All vehicles move with their initial velocity during $0 < t < t_d$. Lane changes are permitted at $t = t_d$ and every 0.05 s thereafter. Other than this initial time, only a few vehicles (usually just 1) or none change lanes at each interval. Hence an interval of 0.05 s is considered adequate. Decreasing the interval would not substantially increase the number vehicles changing lanes.

In Fig. 2, the average velocity of cars 500–524 is shown as a function of time; the black line is for those in lane 1 and the gray line for those in lane 2. Starting velocities were 27.0 and 22.1 m/s ($h = 40$ and 30 m), respectively. Initial probabilities of occupancy were $p_1 = p_2 = 0.5$ and the expansion factors were $e_1 = e_2 = 1.1$. Since the velocity of the lead vehicle was 33 m/s, an increase in average velocity near 300 s was observed. Velocity synchronization occurred within approximately 100 s. Even for large time intervals for lane changing (as much as 5 s), synchronization took place in about the same amount of time.

The car number of the vehicle(s) changing lanes at time t is displayed in Fig. 3. The total number of lane changes was 988 (801 after $t = 0.75$ s) in 500 s. The "upper diagonal" pattern indicates that lane changes are frequent in the early stages of synchronization but become less frequent as it is established and as the transition to higher speeds moved up-

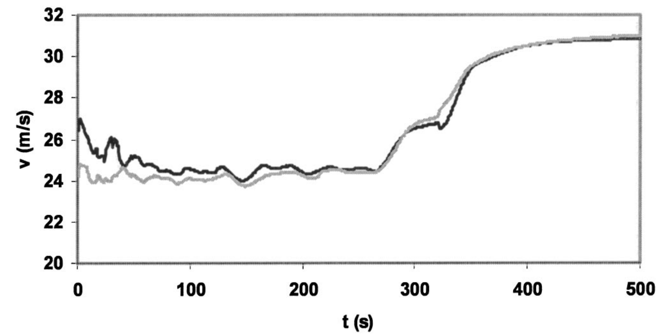

FIG. 2. Average velocity of cars 500–524 in lane 1 (black) and lane 2 (gray) vs time. Starting velocities were 27.0 and 22.1 m/s, respectively. Initial probabilities of occupancy were $p_1 = p_2 = 0.5$ and expansion factors were $e_1 = e_2 = 1.1$. The velocity of the lead vehicle was 33 m/s, which produces an increase in velocity near 300 s. The delay time was $t_d = 0.75$ s. Velocity synchronization occurred within approximately 100 s.

stream. Although average velocities in the two lanes are nearly the same, this is not considered a transition to the SF phase because the velocities remain rather high and the flow is not congested.

IV. ON-RAMP MODEL

Simulations to demonstrate the transition FF to SF are presented in this section. To simplify, let us consider a single lane (lane 1) with an on-ramp (lane 2) as shown in Fig. 4. In a region $-d_{merge} < x < 0$ vehicles may merge from lane 2 to lane 1. In addition to the rules of Sec. III, applied to changes from lane 2 to lane 1 only, merging is permitted if the headways to the leading vehicle, d, and the trailing vehicle, b (both in lane 1), are larger than the safe distances d_{safe} and b_{safe} determined by the optimal velocity function and the candidate velocity v and the trailing vehicle velocity u, respectively, where $v = V(d_{safe})$ and $u = V(b_{safe})$. If the trailing vehicle is in lane 2, only $d > d_{safe}$ is required. (All quantities are evaluated at the delayed time $t - t_d$.) With this geometry SF refers to the single-lane equivalent of synchronized flow for multilane highways. The SF phase is characterized by its region of the two-dimensional vehicle density-flow rate space.

In simulations at low vehicle density, it has been found

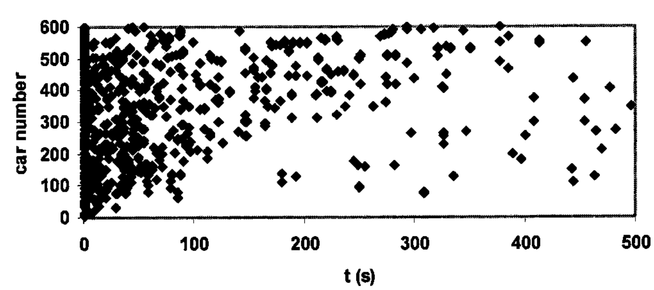

FIG. 3. Car number of lane changes vs time of change. Lane changes were calculated in random order every 0.05 s for each vehicle (on average). The total number of lane changes was 988 (801 after 0.75 s) in 500 s.

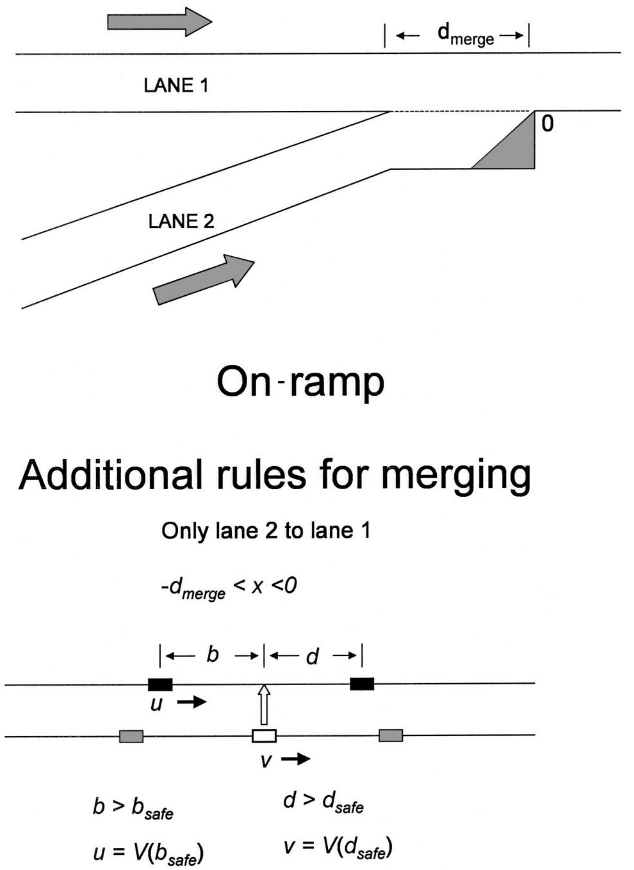

FIG. 4. Geometry of an on-ramp for vehicles to enter a single-lane highway in the on-ramp model. The distance over which vehicles may merge is denoted d_{merge}. Only lane changes from the on-ramp (lane 2) to lane 1 are permitted. Additional (to those allowed in Fig. 1) merges are permitted if the headways to the leading vehicle, d, and the trailing vehicle, b, in lane 1 are larger than the safe distances d_{safe} and b_{safe} determined by the optimal velocity function and the candidate velocity v and the trailing vehicle velocity u, respectively. All quantities are evaluated at the delayed time $t-t_d$.

that vehicles often maintain the same velocity as the preceding vehicle, even if it is a large distance away. In some simulations with an on-ramp, this defect has led to vehicles essentially stalling on lane 2 because the first in line has reached the end of the merge region without merging. To remedy this defect, the following modification was made. When the headway is large, the desired velocity is

$$V_{desired}=\alpha v_{n-1}(t-t_d)+(1-\alpha)V_{OV}, \tag{5}$$

where

$$\alpha=\exp(1-\Delta/L), \quad \Delta>L, \tag{6}$$

and

$$\Delta=\Delta x_n(t-t_d)+t_d\Delta v_n(t-t_d), \tag{7}$$

provided $V_{OV}>v_n(t)$. A suitable value for L is 100 m. L corresponds to the "synchronization distance" first introduced in Refs. [8] and [11]. In the present model, if the nth

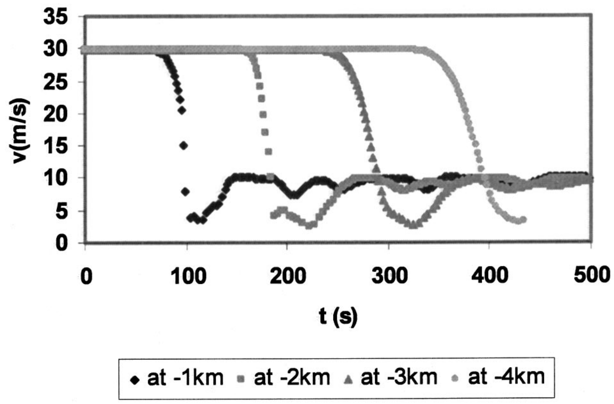

FIG. 5. (Color online) Velocity of vehicles in lane 1 passing points 1, 2, 3, and 4 km upstream of the end of the merge region ($x=0$) vs time. The beginning of the merge region was at $x=-d_{merge}=-2$ km. The transition to the equivalent of synchronized flow for a single-lane highway occurred at approximately 100 s for the -1 km position and moved upstream 1 km every 100 s. The initial headway was $h=40$ m, $p_1=1$, $p_2=0.9$, $e_1=1.22$, and $e_2=1.1$. The lead vehicle velocity was 33 m/s.

vehicle is within L of the preceding vehicle (labeled $n-1$), it attempts to adopt the velocity v_{n-1} provided $v_n<V_{OV}$ [Eqs. (2) and (3)]. If the headway exceeds L, the nth vehicle attempts to adopt the velocity $V_{desired}$ [Eq. (5)], which is in contrast to the model of Refs. [8] and [11] where it would attempt to adopt the safe speed.

In Fig. 5 the velocity of vehicles in lane 1 passing $x=-1$, -2, -3, and -4 km as a function of time is shown. The width of the merge region is $d_{merge}=2$ km, so the -1 km point is within this region. The transition to SF occurred at approximately 100 s for the -1 km position and moved upstream at a speed of 10 m/s. Typically, the velocity of downstream fronts is only 5 m/s or less [2]. Since no parameters were adjusted in this calculation, I consider obtaining this velocity to within a factor of 2 satisfactory. Increasing τ and t_d reduces the velocity somewhat, although to obtain a realistic velocity requires that a small jam form at beginning of the SF phase.

The downstream edge of the SF was pinned at the on-ramp, whereas the upstream front progressed in both lanes beyond the merge region. The profile within the SF changes somewhat with time, unlike the flat profile found in a typical jam. Different random sequences of merge attempts gave essentially the same upstream front, but the profile within the SF varied. In this simulation, the initial headway was taken to be $h=40$ m for both lanes, $p_1=1$, $p_2=0.9$, $e_1=1.22$, and $e_2=1.1$. The lead vehicle velocity was kept at 33 m/s. The results in Fig. 5 are qualitatively similar to Fig. 3a of Ref. [8] and Figs. 8c and 13b of Ref. [11], computed with a different model.

The pattern of merges (the car number of the vehicle merging as a function of time) in Fig. 6 is different from that of Fig. 3 where only synchronization of velocity occurs. The steeper diagonal line corresponded to vehicles in lane 2 (the on-ramp) reaching $x=-d_{merge}$. The remaining band of points resulted from merges associated occurring near the end of

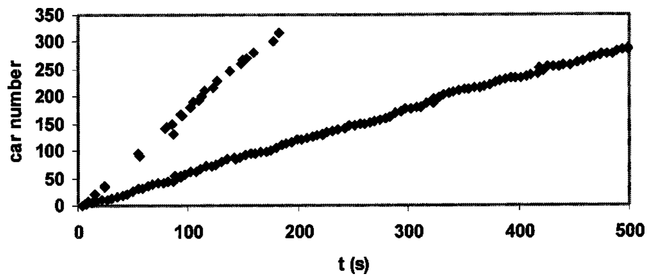

FIG. 6. Car number of merges vs time. Parameters are the same as in Fig. 5.

the on-ramp at $x=0$. Most merges take place near $x=0$ at low velocity.

The most compelling evidence that the SF has been established is given in Fig. 7. Here the 20-car average flow rate for lane 1 measured at $x=-2$ and -3 km is plotted against vehicle density. The cluster of points near (0.05/m, 0.4/s) is indicative of SF—flow rates at high density comparable to free-flow rates at much lower density. The curves of points running from left to right are due to FF in the beginning transforming to SF. For comparison, the density-rate curve given by the OVF, where rate$=V(h)/h$ and density$=1/h$, is shown as the solid line. Note that the SF cluster is near the OVF curve at a density higher than the peak-flow density ≈ 0.03/m. At $x=25$ m (immediately downstream of the merge region), free flow was reestablished at lower density near the linear portion of the OVF curve. These results are similar to those in Figs. 8c and 13b in Ref. [11] obtained by other means.

Near an on-ramp, the velocity pattern depends on delay time (see Fig. 8). At $x=-3$ km, SF is formed in the range of delay 0.7 s$<t_d<$0.85 s. For $t_d>$0.9 s the congestion is sufficiently strong that a jam forms. This seems reasonable because the larger the delay, the less stable the system is and the more likely jams will form. Not only does the average velocity of vehicles in the congested region decrease, the width of the transition front also decreases with increasing delay time—both characteristic of jam formation. The distinction between SF at low velocities and jam formation therefore appears somewhat arbitrary in this situation. It can be seen that the transition from FF occurs markedly later as the delay time increases. No other simulations in the literature have demonstrated the dependence (of the transition from FF) on driver reaction time.

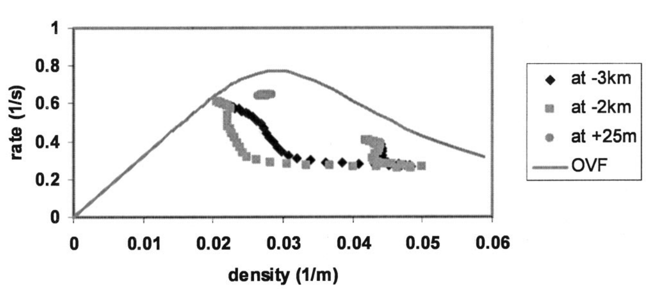

FIG. 7. (Color online) Flow rate for lane 1 vs vehicle density measured at 25 m beyond the end of the merge region, at -2 and -3 km. The rate-density curve given by the optimal velocity function (OVF) is the solid line. The transition from free flow to synchronized flow [near (0.05/m, 0.4/s)] can be seen for -2 and -3 km. At $+25$ m, free flow is reestablished.

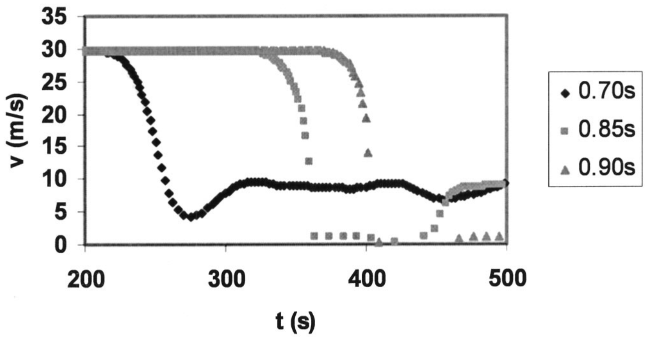

FIG. 8. (Color online) Velocity at $x=-3$ km vs time for various delay times t_d=0.7, 0.85, and 0.9 s. Parameters are the same as in Fig. 5. Vehicles from only lane 1 are shown.

In simulations not shown, the velocities of vehicles at $x=-d_{merge}$ were found to have similar time dependences for merge regions of different size (1–3 km).

V. FURTHER DISCUSSION AND CONCLUSIONS

According to Kerner [2], transitions to the WMJ phase often follow transitions into SF. In the present on-ramp simulations SF to WMJ was not observed in actual time, perhaps because of the limited number of vehicles (600) or the limited amount of simulated time (500 s). Possibly the restriction to a single-lane highway, rather than a multilane highway, with on-ramp makes it more difficult to observe SF to WMJ transitions in simulations.

The on-ramp simulations considered in this paper are for high flow rates in both lanes. The incoming rates are sufficiently large to be in the regime of the "general pattern" of Kerner [2] or the "homogeneous congested state" of Helbing *et al.* [20]. The total of the incoming flux of vehicles in both lanes exceeds the capacity of the single lane beyond the merge region.[1] To conserve the number of vehicles flow is reduced in each lane in the merge region with the concomitant formation of jams or SF moving upstream. The velocity of the upstream front was found to be 10 m/s, which is at least twice that of the experimental value [2]. Since no parameters of the model were adjusted, but were determined from other considerations ahead of time, this discrepancy is not considered serious.

Deciding between the descriptions of traffic phases due to Kerner [2] and to Helbing [1] is beyond the scope of the present simulations, which have been done on a laptop computer. Substantially larger number of vehicles (10^4 or 10^5) and run times of hours might be needed to examine these issues.

The results of the calculations given in this paper can be compared to on-ramp simulations done by Berg and Woods

[1]The single-lane capacity is less than the maximum OVF flow, 0.77 vehicles/s, because the velocity is not optimal.

[21] using the original OV model. They found the phases observed by Helbing using a nonlocal, gas-kinetic-based model [20], but did not observe a phase specifically identified as synchronous flow. (There may be correspondence between the homogenous congested phase and synchronous flow, however [20].) One can begin to understand these differences by considering the steady-state or equilibrium solutions of the original OV model and the modified OV model.

The steady-state solutions of the present model [Eqs. (1)–(7)] are $v_n = v_{n-1}$ if $v_n \leqslant V_{OV}(\Delta x_n)$ and $\Delta x_n < L$. This implies that a steady-state solution can be anywhere on or below the OVF curve in the two-dimensional flow-density space shown in Fig. 7 (except for a small region where the density does not exceed $1/L$). The steady-state solutions of the original OV model, however, can only be on the OVF curve—that is, on the fundamental diagram where $v_n = v_{n-1}$ and $v_n = V_{OV}(\Delta x_n)$. The OV model as modified in this paper satisfies a basic requirement (postulate) of the three-phase model; namely, the steady-state solutions must cover a two-dimensional region of flow-density space [4,5]. The steady-state solutions of the original OV model cover only a one-dimensional space, the OVF curve.

In summary, a dual-lane calculation has shown velocity synchronization due to lane changing in a free-flow state. The existence and, perhaps more importantly, the formation of the single-lane equivalent of the synchronized flow state have been demonstrated in on-ramp simulations. For delays typical of drivers (~ 0.75 s), stable transitions to synchronized flow have been observed. For long delays ($\geqslant 1$ s), jam formation, rather than synchronized flow, is found as expected.

The pattern of lane changes, specifically car number versus time of lane change, has been found to be useful for detecting transitions.

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